Exercise 59

Find an equation of the tangent to the curve at the given point.

$$y = \sqrt{1 + 4\sin x}, \quad (0, 1)$$

Solution

The aim is to find the slope of the tangent line at $x = 0$. Take the derivative of y.

$$\begin{aligned} y' &= \frac{d}{dx}\sqrt{1 + 4\sin x} \\ &= \frac{1}{2}(1 + 4\sin x)^{-1/2} \cdot \frac{d}{dx}(1 + 4\sin x) \\ &= \frac{1}{2}(1 + 4\sin x)^{-1/2} \cdot (4\cos x) \\ &= \frac{2\cos x}{\sqrt{1 + 4\sin x}} \end{aligned}$$

Plug in $x = 0$.

$$y'(0) = \frac{2\cos 0}{\sqrt{1 + 4\sin 0}} = 2$$

Use the point-slope formula with this slope and the given point $(0, 1)$ to get the equation of the tangent line.

$$\begin{aligned} y - 1 &= 2(x - 0) \\ y - 1 &= 2x \\ y &= 2x + 1 \end{aligned}$$

Below is a graph of the curve and its tangent line at $(0, 1)$.